2024/FYUG/ODD/SEM/
STADSM-201T/151

FYUG Odd Semester Exam., 2024

STATISTICS

(3rd Semester)

Course No. : STADSM-201T

(Random Variables and Probability Distribution)

Full Marks : 70
Pass Marks : 28
Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* from the following : $2\times2=4$

(a) Define continuous random variable and probability density function (p.d.f.).

(b) Define cumulative density function and state any one of its property.

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(*Turn Over*)

(c) If $p(x) = \begin{cases} \dfrac{x}{15} \ ; & x = 1, 2, 3, 4, 5 \\ 0 \ ; & \text{otherwise} \end{cases}$

find $P\{X = 1 \text{ or } X = 2\}$.

2. (a) A random variable X has following probabilities function :

x	0	1	2	3	4	5	6	7
$p(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) Find k.

(ii) Evaluate $P(X < 6)$, $P(X \geq 6)$. 1+2+2=5

(b) A random variable X has the following probability function :

x	-2	-1	0	1	2	3
$p(x)$	0·1	k	0·2	$2k$	0·3	k

Find the value of k and calculate mean and variance. 1+2+2=5

OR

3. (a) The diameter of an electric cable says X is assumed to be continuous random variable with p.d.f.

$$f(x) = 6x(1 - x), \quad 0 \leq x \leq 1$$

(i) Check $f(x)$ is p.d.f.

(ii) Find b such that $P(X < b) = P(X > b)$. 2+3=5

(b) A continuous random variable has a p.d.f. $f(x) = 3x^2$, $0 \leq x \leq 1$. Find a and b such that—

(i) $P(X \leq a) = P(X > a)$;

(ii) $P(X > b) = 0 \cdot 05$. 3+2=5

UNIT—II

4. Answer any *two* from the following : 2×2=4

(a) Define expectation of a random variable and state some of its properties.

(b) Define variance of a random variable and state some of its properties.

(c) Define moment-generating function and state some of its properties.

5. (a) State and prove addition theorem of expectation. 5

(b) If X and Y are two random variables and a, b are constants, then find—

(i) $V(a)$, $V(b)$;

(ii) $V(X + Y)$;

(iii) $V(aX + bY)$. 1+2+2=5

OR

6. *(a)* Prove that m.g.f. of the sum of a number of independent random variables is equal to the product of their respective m.g.fs. 5

(b) Let X be a random variable with the following probability distribution :

x :	-3	6	9
$p(x)$:	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find $E(X)$ and $E(X^2)$. 5

UNIT—III

7. Answer any *two* from the following : 2×2=4

(a) Define two-dimensional random variable and joint probability mass function.

(b) Define marginal and conditional distribution function.

(c) Define independence of random variables.

8. Two random variables X and Y have the following joint probability density function

$$f(x, y) = \begin{cases} 2 - x - y; & 0 \le x \le 1, \ 0 \le y \le 1 \\ 0; & \text{otherwise} \end{cases}$$

Find—

(a) marginal p.d.f. of X and Y;

(b) conditional p.d.f. of X and Y;

(c) variance of X. 4+4+2=10

OR

9. *(a)* Define conditional expectation and conditional variance. 2+2=4

(b) Two dice are thrown. Let X_1 be the score on first die and X_2 be the score on second die. Let Y denote maximum of X_1 and X_2, i.e., $Y = \max(X_1, X_2)$.

Write down the joint distribution of Y and X_1. 6

UNIT—IV

10. Answer any *two* from the following : 2×2=4

(a) State the p.m.f. of binomial distribution with parameters n and p, and find its mean.

(b) Find the moment-generating function of Poisson distribution.

(c) State some properties of normal distribution.

11. *(a)* Derive the moment-generating function of binomial distribution with parameters n and p, and hence find its mean and variance. 5

(b) Show that Poisson distribution is a limiting case of binomial distribution. 5

OR

12. *(a)* State and prove the additive properties of binomial distribution. 5

(b) If X and Y are independent Poisson variates, then show that conditional distribution of X given $X+Y$ is binomial. 5

UNIT—V

13. Answer any *two* from the following : 2×2=4

(a) State Chebyshev's inequality.

(b) State Markov's inequality.

(c) What is convergence in probability?

14. *(a)* State and prove weak law of large numbers. 6

(b) A symmetric die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixed. 4

OR

15. *(a)* State and prove Bernoulli's law of large numbers. 6

(b) How large a sample must be taken in order that the probability will be at least 0·95 that $\bar{X}_n$ will be within μ? (μ is unknown and $\sigma = 1$.) 4

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